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Anomaly Freedom in *CP* Violation Preon Model

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Based on a preon model satisfying both the $SU(2)$ and Lorentz invariances, it is shown that *CP* is violated, and anomalies are also eliminated.

Recently, the authors (Sugita *et al.*, 1994; Okamoto *et al.*, 1995) have shown that *CP* is violated in only one family of leptons and quarks, based on the preon model in which particle and antiparticle construct the $SU(2)$ weak isospin doublet.

We have considered a preon $\langle\langle a\rangle\rangle$ with spin 1/2 and electric charge 1/2, a preon $\langle\langle l\rangle\rangle$ with spin 1/2 and electric charge -1, and a preon $\langle\langle q_i\rangle\rangle$ with spin 1/2 and electric charge $-1/3$ having colors of $i = R, G, B$. Then the first family of leptons and quarks is written as

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} a_L \\ a_L^{cp} \end{pmatrix} a_R l_L, \qquad e_R = (a_R^{cp} a_R l_R)$$

$$\begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} = \begin{pmatrix} a_L \\ a_L^{cp} \end{pmatrix} a_R q_{iL}, \qquad u_{iR} = (a_R a_R q_{iR}), \qquad d_{iR} = (a_R^{cp} a_R q_{iR})$$

where the subscripts L and R denote the left- and right-handed particles, respectively, and a_L^{cp} means the left-handed particle operated on by charge conjugation C and then parity transformation P, namely

$$a_L^{cp} \equiv \gamma^0 C \gamma^0 \tfrac{1}{2}(1 - \gamma_5) a^* \tag{1}$$

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Unfortunately, in this model, the anomalies cannot be removed. To eliminate the anomalies, we have to introduce a left-handed preon $\langle\langle f_{jL}\rangle\rangle$ with spin 1/2 and electric charge 1/2 having the integers $j = 1, 2, 3, \ldots$ which determine the number of families of leptons and quarks.

Then the first family of leptons and quarks is composed as follows:

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} a_L \\ a_L^{cp} \end{pmatrix} f_{1L} l_L, \qquad e_R = (a_R^{cp} f_{1L} l_R)$$

$$\begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} = \begin{pmatrix} a_L \\ a_L^{cp} \end{pmatrix} f_{1L} q_{iL}, \qquad u_{iR} = (a_R f_{1L} q_{iR}), \qquad d_{iR} = (a_R^{cp} f_{1L} q_{iR})$$

From this model, we can account for CP violation and anomaly freedom. First we consider CP violation satisfying both the $SU(2)$ and Lorentz invariances. The Lagrangian describing the electroweak interactions is written as

$$L = \bar{\chi}\gamma^{\mu}\left(i\partial_{\mu} - \frac{g}{2}\boldsymbol{\tau}\cdot\mathbf{W}_{\mu}\right)\chi \tag{2}$$

where χ is the weak-isospin doublet, g is the coupling constant, $\mathbf{W}_{\mu}$ are three gauge fields of $SU(2)_L$, and $\boldsymbol{\tau}/2$ are generators of $SU(2)_L$.

Now consider the following weak isospin doublet:

$$\chi = (a_L \, A a_L^{cp})^T \tag{3}$$

where the superscript T means transposed and A is a matrix. To satisfy the $SU(2)_L$ gauge symmetry, the matrix A must satisfy the condition (Sugita *et al.*, 1994)

$$ACA^*C = -I_4 \tag{4}$$

where I_4 is a 4×4 unit matrix. If $A = M\gamma^0$ and M is a Lorentz scalar, equation (2) is invariant under the Lorentz transformation. Then using M, we rewrite equation (4) as

$$\tilde{M}M^* = -I_4 \tag{5}$$

where $\tilde{M} \equiv M(\gamma_5 \to -\gamma_5,\ \gamma^{\mu} \to -\gamma^{\mu *},\ \sigma^{\mu\nu} \to -\sigma^{\mu\nu *})$ with $\sigma^{\mu\nu} = (i/2)[\gamma^{\mu}, \gamma^{\nu}]$. If the matrix M satisfies equation (5), the $SU(2)$ and the Lorentz invariances hold. For example, we can choose $\exp[id_{\mu}\gamma^{\mu}]\gamma_5$, $\exp[id_{\mu\nu}\gamma_5\sigma^{\mu\nu}]\gamma_5$, and $\exp[it_{\kappa\mu\nu}\gamma_5\sigma^{\kappa\mu}\gamma^{\nu}]\gamma_5$ as the matrix M. Here d_{μ} is a real vector in Minkowski space and independent of space.

In a previous paper (Okamoto *et al.*, 1995), as an example, we considered the matrix

$$\begin{aligned} A &= M\gamma^0 \\ &= \exp[id_{\mu}\gamma^{\mu}]\gamma_5\gamma^0 \end{aligned} \tag{6}$$

and showed direct *CP* violation in β-decay of the nucleon satisfying both the $SU(2)$ and Lorentz invariances. That is, Hermite conjugation of the preon tensor $T_{\mu\nu}^{\langle\text{preon}\rangle\dagger}$ is written as

$$T_{\mu\nu}^{\langle\text{preon}\rangle\dagger} = \frac{1}{2} d^{-2} \sin^2 d \operatorname{Tr}\left(\not{k}^\dagger d_\sigma \gamma^{\sigma\dagger} \gamma^0 \gamma_\mu \not{k}' \gamma_\nu d_\rho \gamma^\rho \gamma^0 \frac{1}{2}(1 - \gamma_5)\right) \tag{7}$$

On the other hand, $T_{\mu\nu}^{\langle\text{preon}\rangle cp}$ is written as

$$T_{\mu\nu}^{\langle\text{preon}\rangle cp} = \frac{1}{2} d^{-2} \sin^2 d \operatorname{Tr}\left(\not{k}^\dagger d_\sigma \gamma^{\sigma} \gamma^0 \gamma_\mu \not{k}' \gamma_\nu d_\rho \gamma^{\rho\dagger} \gamma^0 \frac{1}{2}(1 - \gamma_5)\right) \tag{8}$$

The difference between equations (7) and (8) is that the dagger † operates on γ_σ and γ_ρ. Thus, we obtain $T_{\mu\nu}^{\langle\text{preon}\rangle\dagger} \neq T_{\mu\nu}^{\langle\text{preon}\rangle cp}$ and we can show direct *CP* violation in β-decay of the nucleon satisfying both the $SU(2)$ and Lorentz invariances.

Next we consider anomaly freedom. From the Nishijima–Gell-Mann relation, the electric charge Q of the preon is written as

$$Q = T^3 + \frac{Y}{2} \tag{9}$$

where T^3 is the third component of weak isospin and Y is the hypercharge of the preon. For example, we can assign the hypercharge for each preon as

$$Y_{a_L} = 0, \qquad Y_{a_R} = 1, \qquad Y_{f_L} = 1$$
$$Y_{l_L} = Y_{l_R} = -2, \qquad Y_{q_L} = Y_{q_R} = -2/3 \tag{10}$$

We apply the electroweak interactions to the preon model and calculate the anomalies. The electroweak interactions are the gauge group of $SU(2)_L \times U(1)$, so left and right are not symmetric and there is a problem of the anomalies. The anomalies are expressed by the Jacobian J of the path integral measure (Fujikawa, 1979). If $J \neq 1$, then it is said that the anomalies exist and the theory contradicts quantum field theory. From the Wess–Zumino consistency relations (Wess and Zumino, 1971) the anomalies are uniquely determined by the third-order term of the field. Therefore, we shall consider the triangle loop diagram and calculate the anomalies.

In electroweak interactions, the gauge fields interacting with the preon field are the gauge field $\mathbf{W}_\mu$ in the $SU(2)_L$ group and the gauge field B_μ in $U(1)$ group. The three-vertex interaction is schematically expressed by **WWW**, **WW***B*, **W***BB*, and *BBB*. The expression matrix, which combines the preon and gauge field appearing in the vertex of the loop, is $\boldsymbol{\tau}/2$ for **W** and the hypercharge Y for B. From this, the anomaly from **WWW** is calculated as $\mathrm{tr}(\tau^a\{\tau^b, \tau^c\}) = 2\delta^{bc}\,\mathrm{tr}(\tau^a) = 0$ and the anomaly does not exist. Similarly, the anomaly from **W**BB does not exist, since $\mathrm{tr}(\tau^a) = 0$. Finally, we calculate the anomalies from **WW**B and BBB. The result is

$$\begin{aligned}
\ln J &= \frac{i}{32\pi^2}\int d^4x\, \epsilon^{\mu\nu\alpha\beta}[\mathrm{tr}\{Y^1_{aL}F_{\mu\nu}(W)F_{\alpha\beta}(W)\} \\
&\quad + \mathrm{tr}\{(Y^3_{aL} + Y^3_{fL} + Y^3_{lL} + 3Y^3_{qL} - Y^3_{aR} - Y^3_{lR} - 3Y^3_{qR})F_{\mu\nu}(B)F_{\alpha\beta}(B)\}] \\
&= \frac{i}{32\pi^2}\int d^4x\, \epsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}(Y^3_{fL} - Y^3_{aR})F_{\mu\nu}(B)F_{\alpha\beta}(B) \qquad (11) \\
&= 0
\end{aligned}$$

where $F_{\mu\nu}(W)$ and $F_{\mu\nu}(B)$ denote the strength of the $SU(2)_L$ gauge field and $U(1)$ gauge field, respectively. From this result, we obtain $J = 1$ and anomalies do not exist in the preon electroweak interactions. We have considered only the first family of leptons and quarks. It is easy to show that there are no anomalies in each family.

Thus, we can show the $SU(2)$ and Lorentz invariances, CP violation, and anomaly freedom by taking into account the preon model including the left-handed weak isospin singlet f_{iL} with the number of families of leptons and quarks.

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